

Radians, Arc Length and the Unit Circle

Grade 12 – Precalculus math · Worksheet · Standards: F-TF.A.1, F-TF.A.2, F-TF.A.3, F-TF.A.4

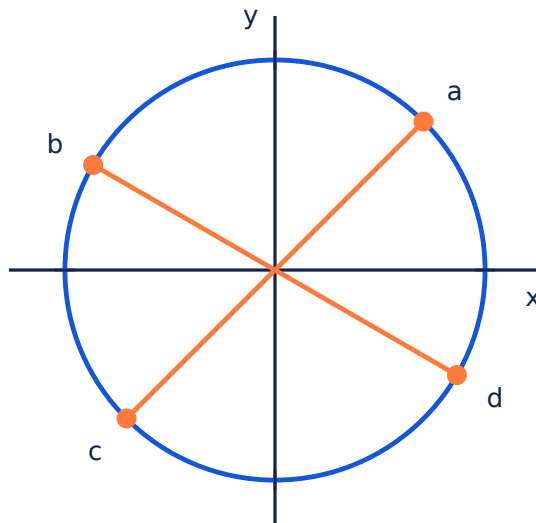
Name: _____ Date: _____ Class: _____

These 20 problems cover radian conversions, arc length, angular and linear speed, exact trigonometric values at special angles, reference angles, symmetry and periodicity. They train standards F-TF.A.1 through F-TF.A.4. The last six problems use Ferris wheels, clocks, bicycles, a lighthouse and a particle moving on the circle.

Standards: F-TF.A.1, F-TF.A.2, F-TF.A.3, F-TF.A.4 · **Level:** Grade 12 Precalculus

Problem 1 Degrees to Radians (easy)

Convert each angle to radians, leaving π in the answer. (a) 45° (b) 150° (c) 225° (d) 330° . The figure shows the four angles.



Four angles drawn on the unit circle.

Problem 2 Radians to Degrees (easy)

Convert each angle to degrees. (a) $2\pi/3$ (b) $7\pi/4$ (c) $3\pi/10$ (d) $5\pi/12$

Problem 3 Arc Length Basics (easy)

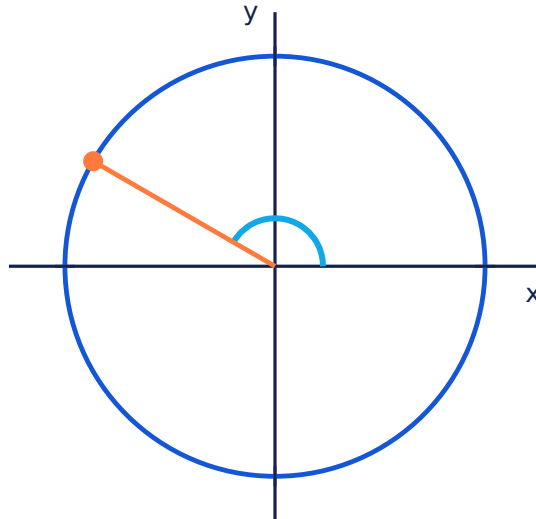
Find the arc length for each central angle. (a) radius 12 inches, angle $5\pi/6$ (b) radius 9 feet, angle $\pi/3$. Give exact answers and decimals to two places.

Problem 4 Special Angle Values (easy)

Give the exact value of each. (a) $\cos(\pi/6)$ (b) $\sin(\pi/3)$ (c) $\tan(\pi/4)$ (d) $\cos(\pi/2)$

Problem 5 Coordinates from the Unit Circle (easy)

The terminal side of $\theta = 5\pi/6$ meets the unit circle as shown. Name the quadrant, the reference angle, and the exact coordinates of the point.



The angle drawn in the figure.

Problem 6 Quadrants and Reference Angles (easy)

Name the quadrant and reference angle of each angle. (a) $4\pi/3$ (b) $7\pi/4$ (c) $5\pi/6$

Problem 7 Exact Values in Every Quadrant (medium)

Find the exact value. (a) $\sin(5\pi/4)$ (b) $\cos(5\pi/6)$ (c) $\tan(7\pi/6)$ (d) $\cos(11\pi/6)$

Problem 8 Negative and Large Angles (medium)

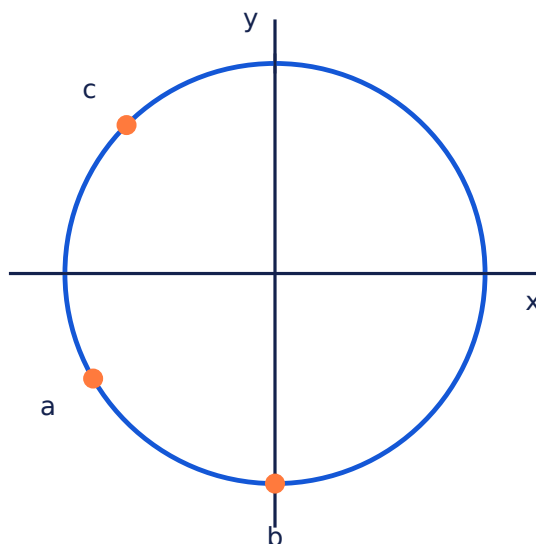
Find the exact value. (a) $\cos(-\pi/3)$ (b) $\sin(-3\pi/4)$ (c) $\sin(13\pi/6)$ (d) $\cos(17\pi/4)$

Problem 9 Even and Odd Relationships (medium)

Suppose $\sin(\pi/8) = a$ and $\cos(\pi/8) = b$. Write each value in terms of a or b . (a) $\sin(-\pi/8)$ (b) $\cos(-\pi/8)$ (c) $\sin(9\pi/8)$ (d) $\cos(7\pi/8)$

Problem 10 Find the Angle from a Point (medium)

Each point lies on the unit circle. Find θ in $[0, 2\pi)$. (a) $(-\sqrt{3}/2, -1/2)$ (b) $(0, -1)$ (c) $(-\sqrt{2}/2, \sqrt{2}/2)$. Then give all angles for part (c). The figure marks points a, b and c in that order.



The three points of the problem.

Problem 11 Find Cosine and Tangent from Sine (medium)

Angle θ lies in Quadrant II and $\sin \theta = 3/5$. Find $\cos \theta$ and $\tan \theta$.

Problem 12 Angle from Arc Length (medium)

An arc of length 15 centimeters lies on a circle of radius 6 centimeters. Find the central angle in radians and in degrees (nearest tenth).

Problem 13 Wheel Speeds (medium)

A wheel of radius 14 inches turns 240 revolutions per minute. Find its angular speed in radians per minute and the linear speed of a point on the rim in miles per hour (63,360 inches per mile).

Problem 14 Coterminal Angles (medium)

Give two positive angles and one negative angle that are coterminal with $5\pi/6$.

Problem 15 Tangent Values (medium)

Find the exact value, or write undefined. (a) $\tan(2\pi/3)$ (b) $\tan(5\pi/4)$ (c) $\tan(-\pi/6)$ (d)

$$\tan(3\pi/2)$$

Problem 16 Ferris Wheel Travel (hard)

A Ferris wheel of radius 25 feet completes one turn every 4 minutes. Find its angular speed in radians per second, the distance a rider travels in 50 seconds, and the rider's linear speed in feet per second.

Problem 17 Minute Hand Tip (hard)

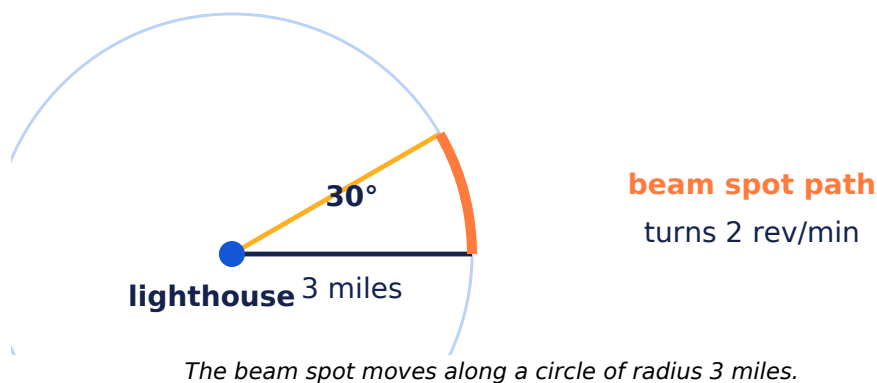
The minute hand of a tower clock is 9 inches long. How far does its tip travel in 20 minutes? In 50 minutes?

Problem 18 Bicycle Wheel Rotation (hard)

A bicycle with 27-inch diameter wheels travels at 12 miles per hour. Find the angular speed of a wheel in radians per minute and in revolutions per minute.

Problem 19 Lighthouse Beam (hard)

A lighthouse beam rotates 2 revolutions per minute. A ship sits 3 miles from the lighthouse. How fast does the spot of light move across the water in miles per minute? How far along its circular path does the spot move while the beam turns 30° ?

**Problem 20 A Particle on the Unit Circle** (hard)

A particle starts at $(1, 0)$ and moves counterclockwise around the unit circle at $3\pi/4$ radians per second. Find its exact position after 2 seconds and after 5 seconds, and the time of its first return to $(1, 0)$.