

Working with Vectors in the Plane

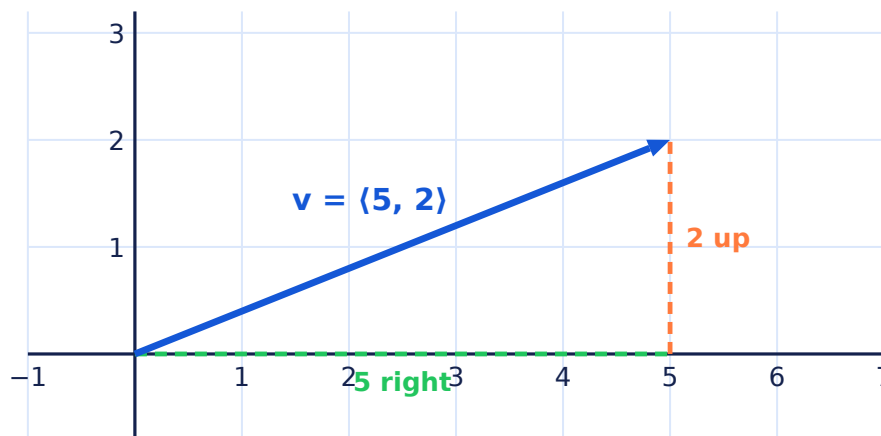
Grade 12 – Precalculus math · Lesson · Standards: N-VM.A.1, N-VM.A.2, N-VM.A.3, N-VM.B.4, N-VM.B.5

Quick answer: A vector describes a displacement or force by its length and direction, recorded as the component pair (a, b) . Its magnitude is the square root of $a^2 + b^2$, vectors add component by component, and the dot product $u \cdot v = u_1v_1 + u_2v_2$ equals $|u||v| \cos \theta$, so it gives the angle between vectors and is zero for perpendicular vectors.

Standards: N-VM.A.1, N-VM.A.2, N-VM.A.3, N-VM.B.4, N-VM.B.5 · **Level:** Grade 12 Precalculus

What is a vector and how do you describe it?

A vector has both magnitude (length) and direction, unlike a scalar, which has only size. In component form a vector is $\mathbf{v} = \langle a, b \rangle$, showing the horizontal and vertical change. From an initial point (x_1, y_1) to a terminal point (x_2, y_2) , the components are $\langle x_2 - x_1, y_2 - y_1 \rangle$.



Vector $\mathbf{v} = \langle 5, 2 \rangle$: $|\mathbf{v}| = \sqrt{29} \approx 5.39$, direction $\approx 21.8^\circ$.

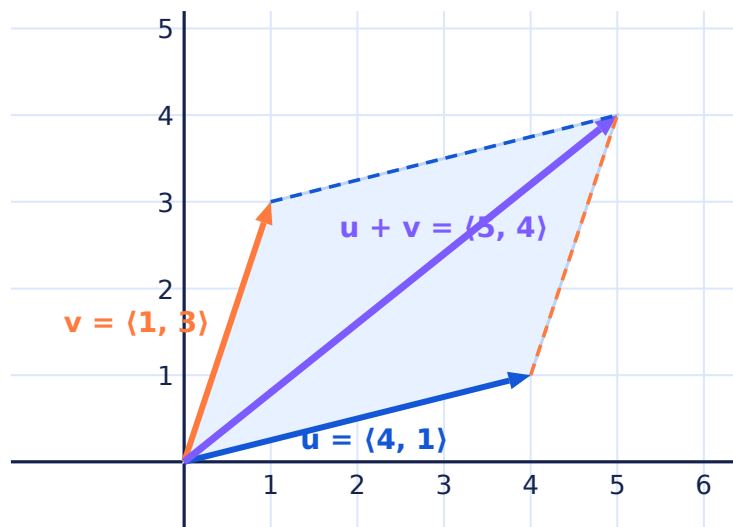
Magnitude and direction

- Magnitude: $|\mathbf{v}| = \sqrt{a^2 + b^2}$.
- Direction angle: $\tan \theta = b/a$, adjusted for the quadrant.
- From magnitude m and angle θ : $\langle m \cos \theta, m \sin \theta \rangle$.

How do you add, subtract and scale vectors?

Add and subtract components: $\langle a, b \rangle \pm \langle c, d \rangle = \langle a \pm c, b \pm d \rangle$. Geometrically, place the vectors tail to tip, or use the parallelogram rule. A scalar multiple $k\mathbf{v} = \langle ka, kb \rangle$ has length $|k| |\mathbf{v}|$ and reverses

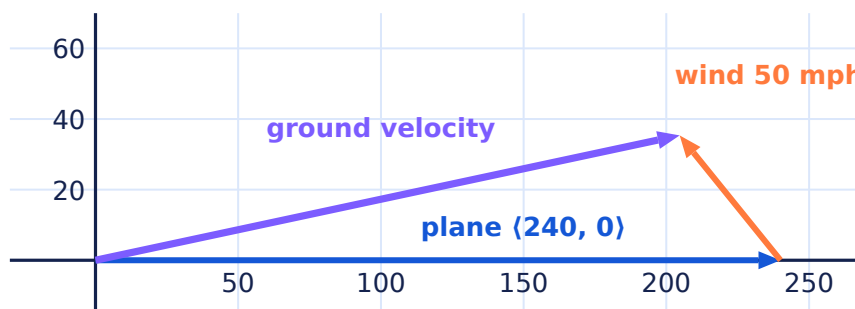
direction if $k < 0$. The unit vector in the direction of $\mathbf{v}$ is $\mathbf{v}/|\mathbf{v}|$.



The diagonal of the parallelogram is $\mathbf{u} + \mathbf{v} = \langle 5, 4 \rangle$.

Worked example: velocity and wind

A plane flies at $\langle 240, 0 \rangle$ mph (due east) in a 50 mph wind blowing toward the northwest, $\langle -35.4, 35.4 \rangle$. The ground velocity is $\langle 204.6, 35.4 \rangle$, with speed $\sqrt{(204.6^2 + 35.4^2)} \approx \mathbf{207.7 \text{ mph}}$ and direction $\approx \mathbf{9.8^\circ}$ north of east.



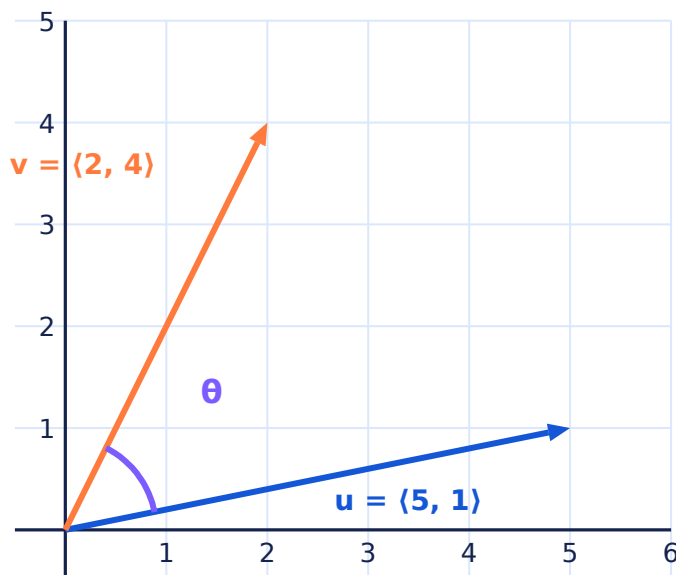
Plane velocity plus wind velocity gives the ground velocity.

What does the dot product tell you?

Method: angle between two vectors

1. Compute $\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2$.
2. Find $|\mathbf{u}|$ and $|\mathbf{v}|$.
3. Use $\cos \theta = \mathbf{u} \cdot \mathbf{v} \div (|\mathbf{u}| |\mathbf{v}|)$, then $\theta = \arccos$ of that value.

For $\mathbf{u} = \langle 5, 1 \rangle$ and $\mathbf{v} = \langle 2, 4 \rangle$, $\mathbf{u} \cdot \mathbf{v} = 10 + 4 = 14$, $|\mathbf{u}| = \sqrt{26}$ and $|\mathbf{v}| = \sqrt{20}$, so $\cos \theta = 14/22.80$ and $\theta \approx \mathbf{52.1^\circ}$. A dot product of 0 means the vectors are perpendicular (orthogonal).



The angle between $\mathbf{u}$ and $\mathbf{v}$: $\cos \theta = 14/(\sqrt{26} \cdot \sqrt{20})$, so $\theta \approx 52.1^\circ$.

Common mistake

Treating the dot product as a vector. It multiplies matching components and adds them, so the result is a single number.

Mathbot's tip

Sketch the vectors before computing. A quick picture tells you whether an angle should be acute or obtuse, which catches sign errors.

Key points to remember

- Components from points: $\langle x_2 - x_1, y_2 - y_1 \rangle$; magnitude $\sqrt{a^2 + b^2}$.
- From magnitude m and angle θ : $\langle m \cos \theta, m \sin \theta \rangle$.
- Add vectors by adding components; scalar multiplication scales every component.
- Unit vector: $\mathbf{v}/|\mathbf{v}|$, a vector of length 1 in the same direction.
- Dot product: $\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 = |\mathbf{u}||\mathbf{v}| \cos \theta$, and $\mathbf{u} \cdot \mathbf{v} = 0$ means perpendicular.

Working with Vectors in the Plane: frequently asked questions

What is the difference between a vector and a scalar?

A scalar is a single number, such as a temperature or a speed of 60 mph. A vector has a size and a direction, such as a velocity of 60 mph toward the northeast, and is written with components.

How do you find the magnitude and direction of a vector?

Use the Pythagorean Theorem for the magnitude, $\sqrt{a^2 + b^2}$, and the inverse tangent of b/a for the

direction angle. Check the quadrant, because a calculator's arctan only returns angles between -90° and 90° .

How do you add two vectors?

Add the horizontal components and add the vertical components. Geometrically, place the tail of the second vector at the tip of the first, and the sum goes from the first tail to the second tip.

What does the dot product tell you?

It measures how much two vectors point the same way. Its value equals $|u||v| \cos \theta$, so it can be used to find the angle between vectors. A dot product of zero means the vectors are perpendicular.

How do you find a unit vector?

Divide the vector by its magnitude. For $\langle 3, 4 \rangle$ the magnitude is 5, so the unit vector is $\langle 3/5, 4/5 \rangle$. The result has length 1 and points in the original direction.