

Working with Vectors in the Plane

Grade 12 – Precalculus math · Test Answer Key · Standards: N-VM.A.1, N-VM.A.2, N-VM.A.3, N-VM.B.4, N-VM.B.5

Each solution shows the work with a scoring note. Award points for correct components, for the correct operation and for the final value separately, so a sign slip costs only one point.

Standards: N-VM.A.1, N-VM.A.2, N-VM.A.3, N-VM.B.4, N-VM.B.5 · **Level:** Grade 12 Precalculus

Question 1 Vector Between Points (2 points)

$$\langle 5 - (-2), -1 - 6 \rangle = \langle 7, -7 \rangle.$$

$$\text{Magnitude} = \sqrt{(49 + 49)} = 7\sqrt{2} \approx \mathbf{9.90}.$$

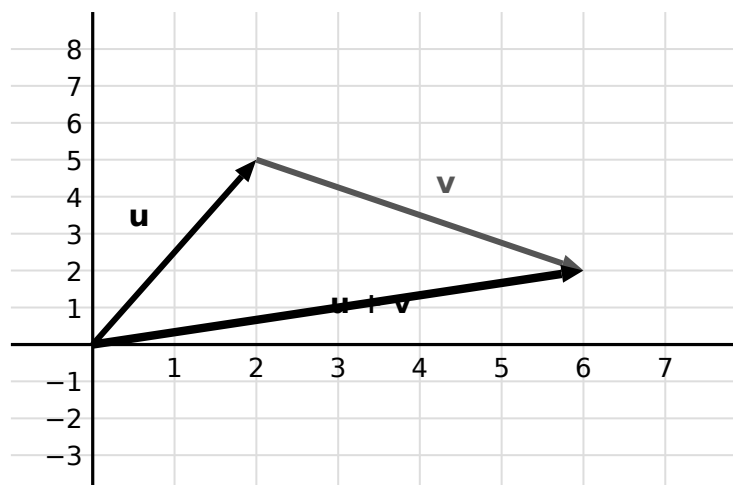
Scoring: 1 point per part.

Question 2 Components from a Direction (2 points)

Horizontal: $40 \cos 125^\circ \approx \mathbf{-22.94}$. Vertical: $40 \sin 125^\circ \approx \mathbf{32.77}$. The horizontal component is negative because 125° is in Quadrant II.

Scoring: 1 point per component.

Question 3 Vector Sum from a Figure (2 points)



The sum goes from the first tail to the second tip.

a. $\mathbf{u} = \langle 2, 5 \rangle$ and $\mathbf{v} = \langle 6 - 2, 2 - 5 \rangle = \langle 4, -3 \rangle$, so $\mathbf{u} + \mathbf{v} = \langle 6, 2 \rangle$.

b. $2\mathbf{u} - \mathbf{v} = \langle 4, 10 \rangle - \langle 4, -3 \rangle = \langle \mathbf{0}, \mathbf{13} \rangle$.

Scoring: 1 point per part.

Question 4 Unit Vector (3 points)

$|\mathbf{v}| = \sqrt{58} \approx 7.616$.

(a) Unit vector = $\langle -3/\sqrt{58}, 7/\sqrt{58} \rangle \approx \langle \mathbf{-0.394}, \mathbf{0.919} \rangle$.

(b) $9/58 + 49/58 = 58/58 = \mathbf{1}$, so the magnitude is 1.

(c) 20 times the unit vector $\approx \langle \mathbf{-7.88}, \mathbf{18.38} \rangle$.

Scoring: 1 point per part.

Question 5 Dot Product and Angle (3 points)

(a) $\mathbf{u} \cdot \mathbf{v} = 5(3) + (-2)(5) = 15 - 10 = \mathbf{5}$.

(b) $|\mathbf{u}| = \sqrt{29} \approx \mathbf{5.385}$, $|\mathbf{v}| = \sqrt{34} \approx \mathbf{5.831}$.

(c) $\cos \theta = 5 \div (5.385 \times 5.831) \approx 0.1592$, so $\theta \approx \mathbf{80.8^\circ}$.

Scoring: 1 point per part.

Question 6 Kayak in a Current (3 points)

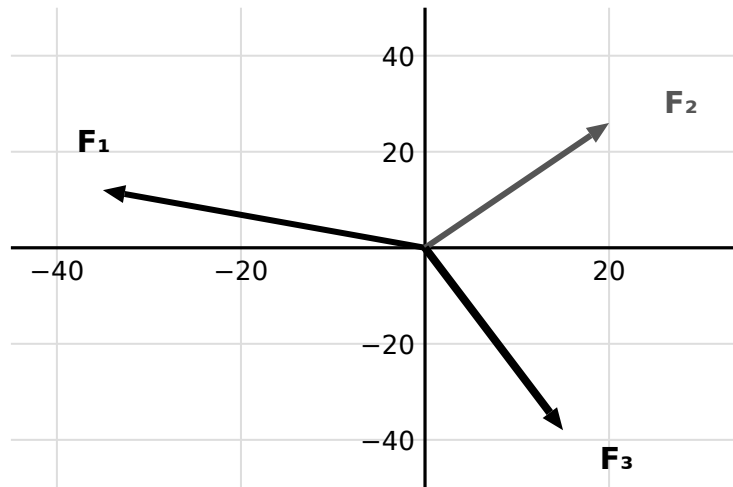
a. With east as +x and north as +y: $\langle \mathbf{1.8}, \mathbf{5.5} \rangle$.

b. $\sqrt{(1.8^2 + 5.5^2)} = \sqrt{33.49} \approx \mathbf{5.79 \text{ mph}}$.

c. $\arctan(1.8/5.5) \approx \mathbf{18.1^\circ}$.

Scoring: 1 point per part.

Question 7 Forces in Equilibrium (3 points)



The third force points opposite to the sum of the other two.

- a. $F_1 + F_2 = (-15, 38)$. Equilibrium requires a total of $\langle 0, 0 \rangle$, so $F_3 = \langle 15, -38 \rangle$.
 b. $|F_3| = \sqrt{(225 + 1444)} = \sqrt{1669} \approx \mathbf{40.9 \text{ lb}}$.

Scoring: 2 points for F_3 (1 for the sum, 1 for the opposite); 1 point for the magnitude.

Question 8 Perpendicular Vectors (2 points)

Perpendicular vectors have dot product 0: $6k + (-5)(4) = 0$, so $6k - 20 = 0$.

$$k = 20/6 = \mathbf{10/3}.$$

Scoring: 1 point for the dot-product equation; 1 point for k .