

Working with Vectors in the Plane

Grade 12 – Precalculus math · Worksheet Answer Key · Standards: N-VM.A.1, N-VM.A.2, N-VM.A.3, N-VM.B.4, N-VM.B.5

Each answer shows the components, the operation and the final vector or number in bold. Sketch each vector to check that the sign and direction make sense before trusting a calculator angle.

Standards: N-VM.A.1, N-VM.A.2, N-VM.A.3, N-VM.B.4, N-VM.B.5 · **Level:** Grade 12 Precalculus

Problem 1 Component Form from Two Points

Subtract the initial point from the terminal point: $\langle 4 - (-3), 1 - 5 \rangle = \langle \mathbf{7}, \mathbf{-4} \rangle$.

Magnitude = $\sqrt{7^2 + (-4)^2} = \sqrt{65} \approx \mathbf{8.06}$.

Problem 2 Magnitude and Direction Angle

(a) $|\mathbf{v}| = \sqrt{49 + 49} = 7\sqrt{2} \approx \mathbf{9.90}$. The point is in Quadrant II with reference angle 45° , so $\theta = \mathbf{135^\circ}$.

(b) $|\mathbf{v}| = \sqrt{97} \approx \mathbf{9.85}$. In Quadrant IV, $\theta = 360^\circ - \arctan(4/9) \approx \mathbf{336.04^\circ}$.

Problem 3 Components from Magnitude and Angle

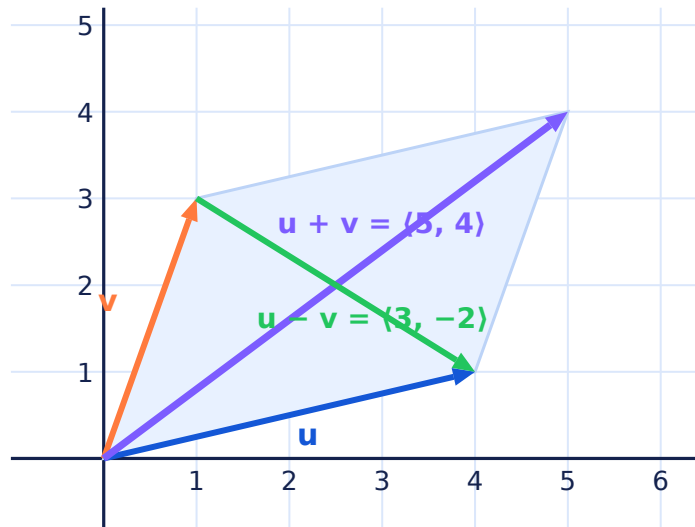
(a) $\langle 18 \cos 40^\circ, 18 \sin 40^\circ \rangle \approx \langle \mathbf{13.79}, \mathbf{11.57} \rangle$.

(b) $\langle 25 \cos 215^\circ, 25 \sin 215^\circ \rangle \approx \langle \mathbf{-20.48}, \mathbf{-14.34} \rangle$. Both components are negative in Quadrant III.

Problem 4 Add and Subtract from a Figure

$\mathbf{u} = \langle 4, 1 \rangle$ and $\mathbf{v} = \langle 1, 3 \rangle$.

$\mathbf{u} + \mathbf{v} = \langle 4 + 1, 1 + 3 \rangle = \langle \mathbf{5}, \mathbf{4} \rangle$. $\mathbf{u} - \mathbf{v} = \langle 4 - 1, 1 - 3 \rangle = \langle \mathbf{3}, \mathbf{-2} \rangle$.



The sum is the long diagonal; the difference joins the tips.

Problem 5 Scalar Multiples

$$5\mathbf{v} = \langle -15, 40 \rangle. \quad -\frac{1}{4}\mathbf{v} = \langle 3/4, -2 \rangle.$$

$$|\mathbf{v}| = \sqrt{73}, \text{ so } |5\mathbf{v}| = 5\sqrt{73} \approx \mathbf{42.72}.$$

Problem 6 Find a Unit Vector

$$|\mathbf{v}| = \sqrt{4 + 25} = \sqrt{29} \approx 5.385.$$

$$\text{Unit vector} = \langle 2/\sqrt{29}, -5/\sqrt{29} \rangle \approx \langle \mathbf{0.371}, \mathbf{-0.928} \rangle. \text{ Its length is 1.}$$

Problem 7 Linear Combination of Vectors

$$2\mathbf{u} = \langle -2, 8 \rangle \text{ and } 3\mathbf{v} = \langle 9, -6 \rangle.$$

$$2\mathbf{u} - 3\mathbf{v} = \langle -2 - 9, 8 + 6 \rangle = \langle \mathbf{-11}, \mathbf{14} \rangle. \text{ Magnitude} = \sqrt{(121 + 196)} = \sqrt{317} \approx \mathbf{17.80}.$$

Problem 8 Angle Between Two Vectors

$$\mathbf{u} \cdot \mathbf{v} = 6(-2) + 1(5) = \mathbf{-7}.$$

$$|\mathbf{u}| = \sqrt{37} \approx 6.083 \text{ and } |\mathbf{v}| = \sqrt{29} \approx 5.385. \cos \theta = -7/(6.083 \times 5.385) \approx -0.2137, \text{ so } \theta \approx \mathbf{102.3^\circ} \text{ (obtuse because the dot product is negative).}$$

Problem 9 Perpendicular Vectors

(a) Set the dot product to 0: $9k - 24 = 0$, so $k = \mathbf{8/3}$.

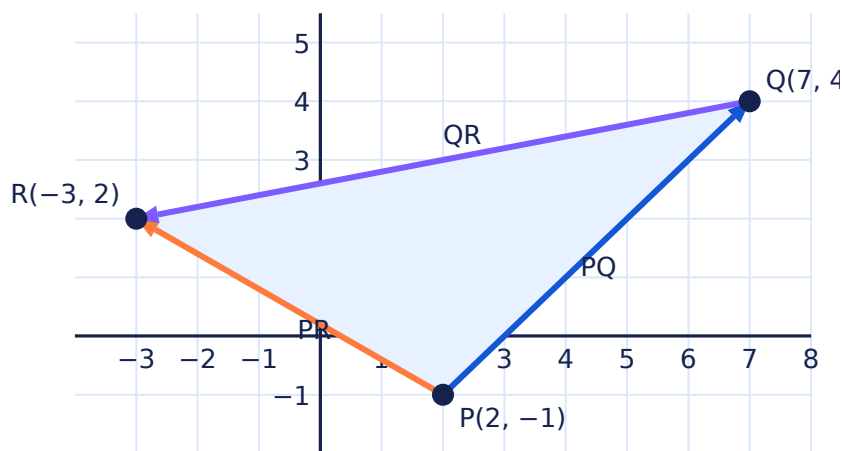
(b) $2(-7) + 7(2) = -14 + 14 = 0$, so **yes**, they are perpendicular.

Problem 10 Vectors Between Three Points

$\mathbf{PQ} = \langle 5, 5 \rangle$, $\mathbf{PR} = \langle -5, 3 \rangle$, $\mathbf{QR} = \langle -10, -2 \rangle$.

$\mathbf{PR} - \mathbf{PQ} = \langle -5 - 5, 3 - 5 \rangle = \langle -10, -2 \rangle = \mathbf{QR} \checkmark$.

Side lengths: $PQ = 5\sqrt{2} \approx \mathbf{7.07}$, $PR = \sqrt{34} \approx \mathbf{5.83}$, $QR = \sqrt{104} \approx \mathbf{10.20}$.



The three side vectors.

Problem 11 Two Forces on a Crate

$F_1 = \langle 80 \cos 20^\circ, 80 \sin 20^\circ \rangle \approx \langle 75.18, 27.36 \rangle$ and $F_2 = \langle 120 \cos 110^\circ, 120 \sin 110^\circ \rangle \approx \langle -41.04, 112.76 \rangle$.

Resultant $\approx \langle \mathbf{34.13}, \mathbf{140.12} \rangle$. Magnitude $\approx \mathbf{144.2 \text{ lb}}$ (check: the forces are 90° apart, so $\sqrt{(80^2 + 120^2)} = 144.2$). Direction $\approx \mathbf{76.3^\circ}$.

Problem 12 Plane and Crosswind

With east as $+x$ and north as $+y$, the plane is $\langle 0, 310 \rangle$ and the wind blows toward the east, $\langle 45, 0 \rangle$. Sum: $\langle \mathbf{45}, \mathbf{310} \rangle$.

Ground speed $= \sqrt{(45^2 + 310^2)} \approx \mathbf{313.2 \text{ mph}}$. Angle east of north $= \arctan(45/310) \approx \mathbf{8.3^\circ}$.

Problem 13 Boat Crossing a River

Velocity = $\langle 3, 7 \rangle$. Speed = $\sqrt{7^2 + 3^2} = \sqrt{58} \approx \mathbf{7.62 \text{ mph}}$. Angle = $\arctan(3/7) \approx \mathbf{23.2^\circ}$ downstream.

Crossing time = $0.8 \div 7 \text{ h} = \mathbf{6.9 \text{ minutes}}$ (the current does not change it). Drift = $3 \times (0.8/7) \approx \mathbf{0.343 \text{ mile}}$.

Problem 14 Force for Equilibrium

The sum of the first two is $\langle 12, 28 \rangle$. Equilibrium needs a total of $\langle 0, 0 \rangle$, so the third force is $\langle \mathbf{-12, -28} \rangle$.

Magnitude = $\sqrt{144 + 784} = \sqrt{928} \approx \mathbf{30.5 \text{ lb}}$.

Problem 15 Diagonals of a Parallelogram

$\mathbf{u} + \mathbf{v} = \langle 3, 5 \rangle$ with magnitude $\sqrt{34} \approx \mathbf{5.83}$. $\mathbf{u} - \mathbf{v} = \langle 7, -3 \rangle$ with magnitude $\sqrt{58} \approx \mathbf{7.62}$.

Check: $34 + 58 = 92$, and $2(26 + 20) = 92 \checkmark$.

Problem 16 Work Done Pulling a Wagon

Force $\langle 45 \cos 30^\circ, 45 \sin 30^\circ \rangle \approx \langle 38.97, 22.5 \rangle$. Displacement $\langle 60, 0 \rangle$.

Work = $38.97(60) + 22.5(0) \approx \mathbf{2,338 \text{ ft}\cdot\text{lb}}$.

Problem 17 Glider in a Wind

A bearing of 060° is a standard angle of 30° , so the glider is $\langle 85 \cos 30^\circ, 85 \sin 30^\circ \rangle \approx \langle 73.61, 42.50 \rangle$. A wind from the north blows toward the south: $\langle 0, -20 \rangle$.

Ground velocity $\approx \langle \mathbf{73.61, 22.5} \rangle$, ground speed $\approx \mathbf{77.0 \text{ mph}}$.

Standard angle $\approx 17.0^\circ$, so the bearing is $90^\circ - 17.0^\circ \approx \mathbf{73.0^\circ}$.

Problem 18 Tension in Two Ropes

Horizontal balance: $T_1 \cos 40^\circ = T_2 \cos 55^\circ$. Vertical: $T_1 \sin 40^\circ + T_2 \sin 55^\circ = 150$.

Substituting $T_2 = T_1 \cos 40^\circ \div \cos 55^\circ$: $T_1(\sin 40^\circ + \cos 40^\circ \tan 55^\circ) = 150$, so $T_1 \approx \mathbf{86.4 \text{ lb}}$ and $T_2 \approx \mathbf{115.3 \text{ lb}}$.

Problem 19 Angle Between Two Trails

Dot product: $8(2) + 3(7) = 37$. Magnitudes: $\sqrt{73} \approx 8.544$ and $\sqrt{53} \approx 7.280$.

$\cos \theta = 37 \div (8.544 \times 7.280) \approx 0.5948$, so $\theta \approx \mathbf{53.5^\circ}$.

Problem 20 Drone Flight Path

$\mathbf{AB} = \langle 9, 7 \rangle$, $\mathbf{BC} = \langle 6, -10 \rangle$, and $\mathbf{AC} = \mathbf{AB} + \mathbf{BC} = \langle \mathbf{15}, \mathbf{-3} \rangle$.

$\mathbf{BA} = \langle -9, -7 \rangle$. $\mathbf{BA} \cdot \mathbf{BC} = -54 + 70 = 16$, with $|\mathbf{BA}| = \sqrt{130}$ and $|\mathbf{BC}| = \sqrt{136}$. $\cos \theta = 16 \div 132.96 \approx 0.1203$, so the angle is $\approx \mathbf{83.1^\circ}$.

Total distance = $\sqrt{130} + \sqrt{136} \approx \mathbf{23.06 \text{ units}}$.